AD-A102 164 NAVAL SURFACE WEAPONS CENTER SILVER SPRING MD F/G 20/5
BANDWIDTH AND FREQUENCY ENHANCEMENT OF FREE ELECTRON LASER IN A--ETC(U)
APR 81 H S UHM
UNCLASSIFIED NSWC/TR-81-103 NL

1 OF 1
AD A
102164

END
DATE
FILMED
8-81
DTIC

AD A102164

NSWC TR 81-103

LEVEL II

(12)

BANDWIDTH AND FREQUENCY ENHANCEMENT OF FREE ELECTRON LASER IN A MILDLY RELATIVISTIC ELECTRON BEAM

BY HAN S. UHM
RESEARCH AND TECHNOLOGY DEPARTMENT

DTIC
ELECTE
JUL 29 1981
S D
E

MARCH 1981

Approved for public release, distribution unlimited

NSWC

NAVAL SURFACE WEAPONS CENTER

Dahlgren, Virginia 22448 • Silver Spring, Maryland 20910

DTIC FILE COPY

81 7 29 027

UNCLASSIFIED

SECURITY CLASSIFICATION OF THIS PAGE (When Data Entered)

REPORT DOCUMENTATION PAGE		READ INSTRUCTIONS BEFORE COMPLETING FORM
1. REPORT NUMBER NSWC TR 81-103	2. GOVT ACCESSION NO. AD-A102164	3. RECIPIENT'S CATALOG NUMBER
4. TITLE (and Subtitle) BANDWIDTH AND FREQUENCY ENHANCEMENT OF FREE ELECTRON LASER IN A MILDLY RELATIVISTIC ELECTRON BEAM		5. TYPE OF REPORT & PERIOD COVERED Final - April 1981
		6. PERFORMING ORG. REPORT NUMBER
7. AUTHOR(s) Han S. Uhm		8. CONTRACT OR GRANT NUMBER(s)
9. PERFORMING ORGANIZATION NAME AND ADDRESS Naval Surface Weapons Center Code R41 White Oak, Silver Spring, MD 20910		10. PROGRAM ELEMENT, PROJECT, TASK AREA & WORK UNIT NUMBERS
11. CONTROLLING OFFICE NAME AND ADDRESS		12. REPORT DATE April 1981
		13. NUMBER OF PAGES 22
14. MONITORING AGENCY NAME & ADDRESS(if different from Controlling Office)		15. SECURITY CLASS. (of this report) UNCLASSIFIED
		15a. DECLASSIFICATION/DOWNGRADING SCHEDULE

16. DISTRIBUTION STATEMENT (of this Report)

Approved for public release, distribution unlimited

17. DISTRIBUTION STATEMENT (of the abstract entered in Block 20, if different from Report)

18. SUPPLEMENTARY NOTES

19. KEY WORDS (Continue on reverse side if necessary and identify by block number)

Microwave Radiation
Electron Beam
Free Electron Laser
Wideband Amplified

gamma con =

20. ABSTRACT (Continue on reverse side if necessary and identify by block number)

The bandwidth and frequency enhancement of the free electron laser instability in a mildly relativistic ($\gamma \lesssim 1.15$) electron beam propagating through a dielectric loaded waveguide is presented. For an appropriate choice of the dielectric constant ε and the thickness of the dielectric material, it is shown that the instability bandwidth and frequency can be greatly enhanced for specified values of the beam energy and the wiggler wavelength.

DD FORM 1 JAN 73 1473 EDITION OF 1 NOV 65 IS OBSOLETE

S/N 0102-014-6601

UNCLASSIFIED

SECURITY CLASSIFICATION OF THIS PAGE (When Data Entered)

UNCLASSIFIED

SECURITY CLASSIFICATION OF THIS PAGE(When Data Entered)

UNCLASSIFIED

SECURITY CLASSIFICATION OF THIS PAGE(When Data Entered)

FOREWORD

The bandwidth and frequency enhancement of the free electron laser instability in a mildly relativistic ($\gamma \lesssim 1.15$) electron beam propagating through a dielectric loaded waveguide is presented. For an appropriate choice of the dielectric constant $\hat{\epsilon}$ and the thickness of the dielectric material, it is shown that the instability bandwith and frequency can be greatly enhanced for specified values of the beam energy and the wiggler wavelength. This research was supported by the Independent Research Fund at the Naval Surface Weapons Center.

H. R. Riedl

H. R. RIEDL
By direction

Accession For		
NTIS GRA&I	X	
DTIC TAB	☐	
Unannounced	☐	
Justification		
By		
Distribution/		
Availability Codes		
Dist	Avail and/or Special	
A		

CONTENTS

ILLUSTRATIONS

One of the most basic instabilities that characterize a relativistic electron beam propagating through a helical (or undulator) wiggler magnetic field is the free electron laser instability.[1-7] In recent years, the free electron laser instability has been extensively investigated with particular emphasis on applications to high power microwave generation. In the previous theoretical studies of this instability, it appears that as a result of the relativistic Doppler effect, the frequency ω of the microwave radiation from the electron beam passing through a wiggler field with the axial wavelength $\lambda_0 = 2\pi/k_0$ is given by $\omega = (k + k_0)\beta c = \gamma^2(1 + \beta)\beta k_0 c$. Here k is the axial wavenumber of the radiation, $\gamma = (1 - \beta^2)^{-1/2}$ is the relativistic factor of the beam electrons, and c is the speed of light in vacuo. In this regard, in order to generate high frequency microwave radiation, high energy beams ($\gamma >> 1$) are required. However, it is very undesirable to have a high γ value in a practical, compact microwave tube. I, therefore, develop a new idea to enhance the frequency upshift without making use of a high γ value. Moreover, I also present a new promising scheme for a broad bandwidth microwave amplifier.

The previous analysis[6,7] by the author for an electron beam in a perfectly conducting waveguide shows that the free electron laser instability is essentially a mode coupling between the electromagnetic and electrostatic modes, whose dispersion relations are expressed as

$$\frac{\omega^2}{c^2} - k^2 = \begin{cases} \alpha_{\ell n}^2/R_c^2 \ , & \text{TE mode} \ , \\ \beta_{\ell n}^2/R_c^2 \ , & \text{TM mode} \ , \end{cases} \tag{1}$$

and

$$\omega = (k + k_0)\beta c \ , \tag{2}$$

respectively, for a tenuous beam. Note that for a tenuous beam, the electrostatic mode can be approximated by the free streaming mode in Eq. (2). In Eq. (1), $\beta_{\ell n}$ and $\alpha_{\ell n}$ are the nth roots of the Bessel function $J_\ell(\beta_{\ell n}) = 0$ and its derivative $J_\ell'(\alpha_{\ell n}) = 0$, respectively, of order ℓ. R_c is the radius of a grounded conducting wall, and TE and TM represent the transverse electric and transverse magnetic modes, respectively. However, for present purposes, I assume a tenuous electron beam propagating through a cylindrical waveguide loaded with a dielectric material in the range $R_w < r < R_c$. Therefore, the radial profile of the dielectric constant is given by $\varepsilon(r) = 1$, for $0 \leq r < R_w$, and $\varepsilon(r) = \hat{\varepsilon}$ for $R_w < r < R_c$. The permeability μ of the dielectric material differs from unity by only a few parts in 10^5, thereby approximating $\mu = 1$ in the subsequent analysis. Cylindrical polar coordinates (r,θ,z) are introduced. In the remainder of this article, properties of the mode coupling of the free electron laser in a dielectric loaded waveguide are investigated, in connection with enhancement of the frequency and bandwidth of the microwave radiation from a mildly relativistic electron beam.

It is, therefore, required to derive the dispersion relation of the transverse electromagnetic mode in a dielectric loaded waveguide. In the analysis, a normal-mode approach is adopted in which all components of the electromagnetic field are assumed to vary according to $\delta\psi(\underset{\sim}{x},t) =$

$\hat{\psi}(r)\exp\{i(\ell\theta + kz - \omega t)\}$, where ℓ is the azimuthal harmonic number. The Maxwell equations for the electromagnetic field amplitudes can be expressed as

$$
\begin{aligned}
\nabla \times \hat{E}(x) &= i(\omega/c)\hat{B}(x) , \\
\nabla \times \hat{B}(x) &= -i(\omega/c)\varepsilon(r)\hat{E}(x) ,
\end{aligned}
\tag{3}
$$

without including the influence of the beam presence. In Eq. (3), $\hat{E}(x)$ and $\hat{B}(x)$ are the electric and magnetic fields. Making use of Eq. (3), it is straightforward to show that

$$\hat{B}_\theta(r) = i\,\frac{\omega\varepsilon(r)}{cp^2}\frac{\partial}{\partial r}\hat{E}_z(r) - \frac{\ell k}{p^2 r}\hat{B}_z(r) , \tag{4}$$

$$\hat{E}_\theta(r) = -i\,\frac{\omega}{cp^2}\frac{\partial}{\partial r}\hat{B}_z(r) - \frac{\ell k}{p^2 r}\hat{E}_z(r) , \tag{5}$$

and

$$\left\{\frac{1}{r}\frac{\partial}{\partial r}r\frac{\partial}{\partial r} - \frac{\ell^2}{r^2} + p^2\right\}\left\{\begin{matrix}\hat{E}_z(r)\\ \hat{B}_z(r)\end{matrix}\right\} = 0 , \tag{6}$$

where $p^2 = \omega^2\varepsilon(r)/c^2 - k^2$, $\hat{E}_\theta(r)$ and $\hat{E}_z(r)$ are the azimuthal and axial components of the electric field and $\hat{B}_\theta(r)$ and $\hat{B}_z(r)$ are the azimuthal and axial components of the magnetic field.

The appropriate boundary conditions of $\hat{E}_z(r)$ and $\hat{B}_z(r)$ at $r = R_c$ are given by $\hat{E}_z(R_c) = [(\partial/\partial r)\hat{B}_z(r)]_{r=R_c} = 0$. Moreover, the fields $\hat{B}(r)$, $\hat{E}_\theta(r)$, and $\hat{E}_z(r)$ are continuous across the boundary $(r = R_w)$ of the dielectric material. Evidently, the solutions to Eq. (6) are given by a linear combination of the Bessel functions of the first kind $J_\ell(pr)$ and the second kind $N_\ell(pr)$ of order ℓ. After a tedious but straightforward algebra, it can be shown that the dispersion relation of the electromagnetic mode in a dielectric loaded waveguide is

expressed as

$$D_T^E(\omega,k)D_T^M(\omega,k) = \frac{\ell^2(\eta^2 - \xi^2)(\eta^2 - \xi^2\varepsilon)}{\eta^4\xi^4} , \tag{7}$$

where the TE and TM dielectric functions are defined by

$$D_T^E = \frac{1}{\eta}\frac{J_\ell'(\eta)N_\ell'(\zeta) - J_\ell'(\zeta)N_\ell'(\eta)}{J_\ell(\eta)N_\ell'(\zeta) - J_\ell'(\zeta)N_\ell(\eta)} - \frac{1}{\xi}\frac{J_\ell'(\xi)}{J_\ell(\xi)} , \tag{8}$$

and

$$D_T^M = \frac{\hat{\varepsilon}}{\eta}\frac{J_\ell'(\eta)N_\ell(\zeta) - J_\ell(\zeta)N_\ell'(\eta)}{J_\ell(\eta)N_\ell(\zeta) - J_\ell(\zeta)N_\ell(\eta)} - \frac{1}{\xi}\frac{J_\ell'(\xi)}{J_\ell(\xi)} , \tag{9}$$

respectively, and the parameters ξ, ζ, and η are defined by

$$\omega^2/c^2 - k^2 = \xi^2/R_w^2 , \tag{10}$$

$$\omega^2\hat{\varepsilon}/c^2 - k^2 = \zeta^2/R_c^2 , \tag{11}$$

and $\eta = \zeta R_w/R_c$, and the prime denotes $J_\ell'(x) = dJ_\ell/dx$ and $N_\ell'(x) = dN_\ell/dx$. Several points are noteworthy from Eqs. (7) - (11). First, the dispersion relations of the TE and TM modes are decoupled for $\ell = 0$. Second, in the limit of $\hat{\varepsilon} \rightarrow 1$ or $R_w/R_c \rightarrow 1$, the dispersion relation in Eq. (7) can be simplified as Eq. (10) with $\xi/R_w = \alpha_{\ell n}/R_c$ for the TE mode and with $\xi/R_w = \beta_{\ell n}/R_c$ for the TM mode. Third, for a completely filled dielectric waveguide ($R_w \rightarrow 0$), Eqs. (7) - (9) can be also reduced to Eq. (11) with $\zeta = \alpha_{\ell n}$ for the TE mode and with $\zeta = \beta_{\ell n}$ for the TM mode.

For given values of the dielectric constant $\hat{\varepsilon}$ and the ratio R_w/R_c, the parameter ξ is determined from Eqs. (7) - (9) in terms of ζ. The oscillation frequency ω and axial wavenumber k in a dielectric loaded waveguide are obtained from the simultaneous solution of Eqs. (10) and (11) for specified ξ and ζ. Figure 1 is a plot of the dielectric

dispersion relation in the parameter space (ω,k) for $\ell = 1$, $R_w/R_c = 0.8$ and several values of the dielectric constant $\hat{\varepsilon}$. The straight lines in Fig. 1 represent the free streaming mode in Eq. (2) for $\gamma = 1.107$ and several values of the normalized wiggler wavenumber k_0R_c. The dispersion curves in Fig. 1 correspond to the lowest radial mode number. In a range of physical parameters, the free streaming mode $\omega = (k + k_0)\beta c$ intersects the dielectric dispersion curve of the electromagnetic mode, thereby indicating the free electron laser instability. The mode coupling occurs at $k = k_p$, distinguishing two cases; (a) the short helical wavelength (SHW) mode corresponding to the normalized mode coupling wavenumber $k_pR_c = 9.3$ for $\hat{\varepsilon} = 2$ and $k_0R_c = 9$ in Fig. 1 and (b) the long helical wavelength (LHW) mode corresponding to $k_pR_c = 12.3$ for $\hat{\varepsilon} = 6$ and $k_0R_c = 1.5$ in Fig. 1.

Short Helical Wavelength Mode. The SHW mode is the high frequency operation of the free electron laser instability. The normalized radiation frequency $\omega/k_0\beta c = (k_p + k_0)/k_0$ versus the dielectric constant $\hat{\varepsilon}$ is plotted in Fig. 2(a) for the SHW mode, $\gamma = 1.1$, several values of k_0R_c, and parameters otherwise identical to Fig. 1. The electromagnetic dispersion relation of a short axial wavelength mode satisfying $kR_c \gg 1$ can be approximated by $\omega \simeq kc/\hat{\varepsilon}^{1/2}$, thereby giving the normalized radiation frequency

$$\omega/k_0\beta c \simeq (1 - \beta\hat{\varepsilon}^{1/2})^{-1}. \tag{12}$$

Shown also in Fig. 2(a) is plot of $\omega/k_0\beta c$ in Eq. (12). Obviously from Eq. (12) and Fig. 2(a), the normalized radiation frequency $\omega/k_0\beta c$ increases rapidly as the dielectric constant $\hat{\varepsilon}$ increases from unity to $\hat{\varepsilon} = 1/\beta^2$. In this regard, it is important to emphasize that the

submillimeter microwave radiation can be easily produced by this scheme even for a moderate electron energy ($\gamma \lesssim 1.15$). The limitation of the radiation frequency is the availability of the proper dielectric material in the present time.

Long Helical Wavelength Mode. After a careful examination of Fig. 1, it is noted that the LHW mode coupling can occur only for the dielectric loaded waveguide. Figure 2(b) is plots of the normalized radiation frequency $\omega/k_0\beta c = (k_p + k_0)/k_0$ versus $\hat{\varepsilon}$ for the LHW mode, $\gamma = 1.15$, several values of k_0R_c, and parameters otherwise identical to Fig. 1. Note that the normalized wiggler wavenumber k_0R_c for the LHW mode is much smaller than that for the SHW mode. However, by an appropriate choice of the dielectric material, the radiation frequency ω for the LHW mode can be many times of the wiggler frequency $k_0\beta c$.

Wide Bandwidth Amplifier. An outstanding microwave amplification requires a broad instability bandwidth. As shown in Fig. 1, the dispersion curves of the free streaming and dielectric waveguide modes for $k_0R_c = 4.43$ and $\hat{\varepsilon} = 4$ coincide practically in the range $4.5 < kR_c < \infty$, thereby indicating possibilities of wide bandwidth amplifier. In general, for a specified beam energy γ, proper choice of the dielectric constant $\hat{\varepsilon}$ and the wiggler wavenumber k_0 gives a wide band free electron laser amplifier. The instability bandwidth can be easily more than fifty percent.

Finally, I conclude this article by pointing out that the instability growth rate for large wavenumber perturbations ($kR_c \gg 1$) is substantially reduced by the axial momentum spread of the beam electrons,[6,7] limiting the enhancement of the bandwidth and radiation frequency. However, the axial momentum spread of an electron beam for the free

electron laser instability can be much less than that for other microwave tubes such as the gyrotron. The growth rate and bandwidth of the free electron laser instability are currently under investigation by the author for a broad range of physical parameters, including the influence of the axial momentum spread on stability behavior.

Acknowledgments. This research was supported by the Independent Research Fund at the Naval Surface Weapons Center.

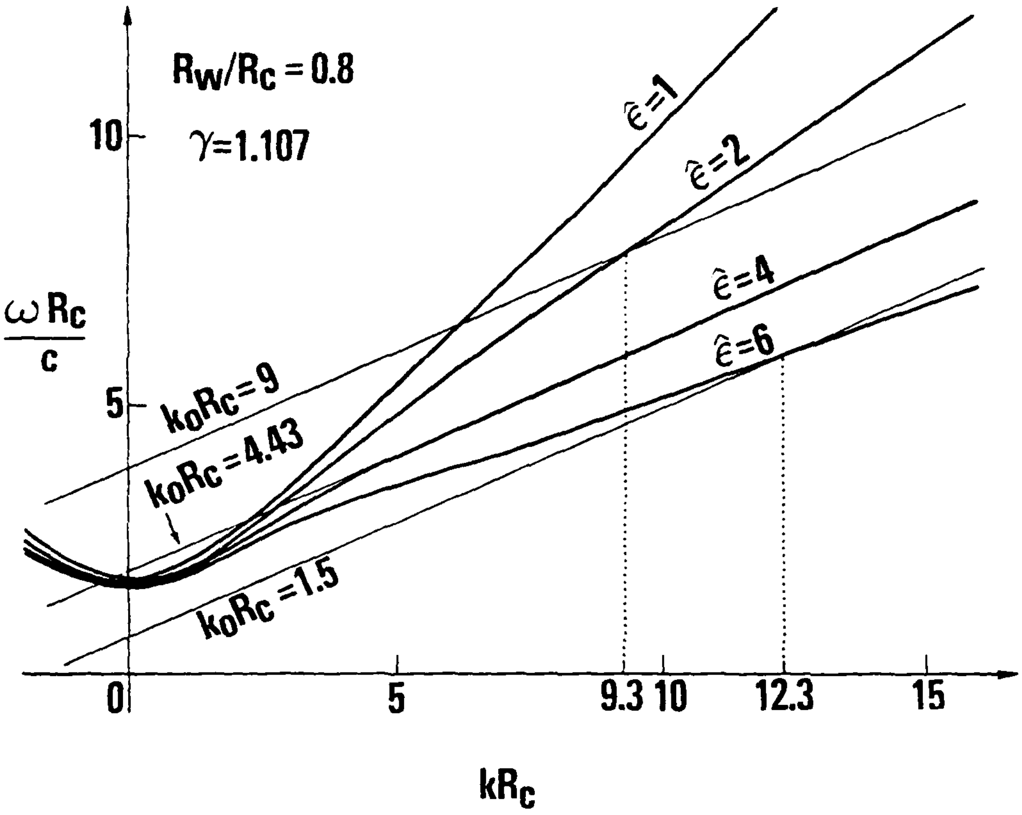

FIGURE 1 PLOT OF THE DIELECTRIC DISPERSION RELATION IN THE PARAMETER SPACE (ω,k) FOR $\ell=1$, $R_w/R_c=0.8$ AND SEVERAL VALUES OF THE DIELECTRIC CONSTANT $\hat{\epsilon}$. FOR $\gamma=1.107$ AND SEVERAL VALUES OF k_0R_c. THE STRAIGHT LINES REPRESENT $\omega=(k+k_0)\beta c$

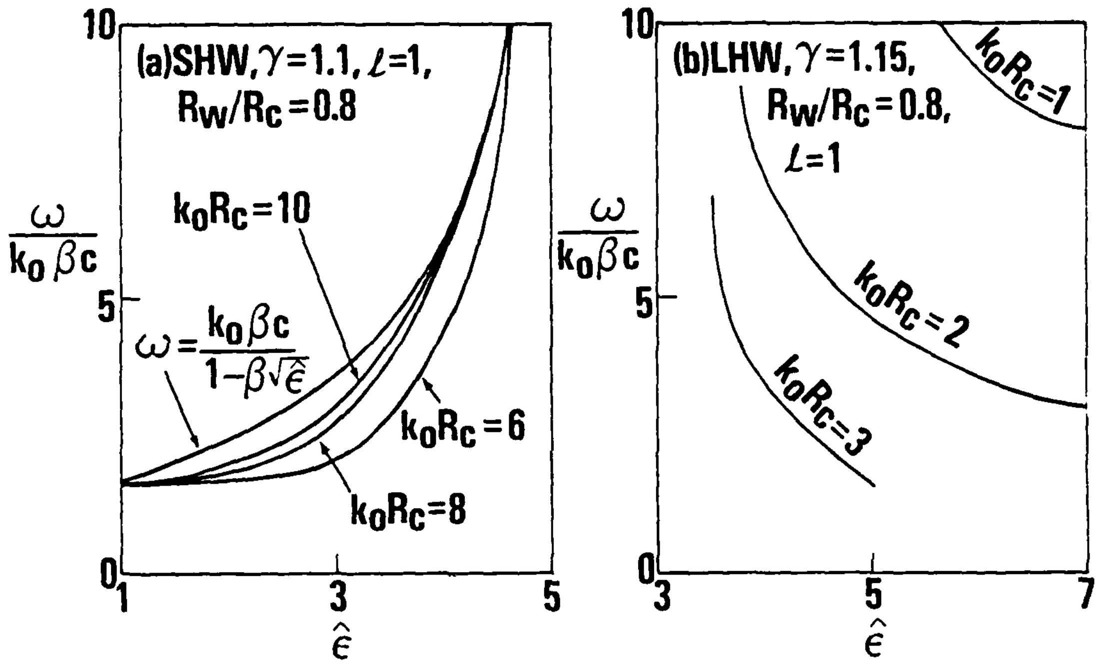

FIGURE 2 PLOT OF NORMALIZED RADIATION FREQUENCY $\omega/k_0\beta c$ VERSUS $\hat{\epsilon}$ FOR (a) THE SHW MODE, γ =1.1, (b) THE LHW MODE, γ =1.15, SEVERAL VALUES OF k_0R_c, AND PARAMETERS OTHERWISE IDENTICAL TO FIG. 1

BIBLIOGRAPHY

1. D. A. G. Deacon, L. R. Elias, J. M. M. Madey, G. J. Ramian, H. A. Schwettman, and T. I. Smith, Phys. Rev. Lett. 38, 897 (1977).
2. P. Sprangle, C. Tang, and W. M. Manheimer, Phys. Rev. Lett. 43, 1932 (1979).
3. A. T. Lin and J. M. Dawson, Phys. Rev. Lett. 42, 1670 (1979).
4. I. B. Bernstein and J. L. Hirshfield, Phys. Rev. A20, 1661 (1979).
5. R. C. Davidson and H. S. Uhm, Phys. Fluids 23, 2076 (1980).
6. H. S. Uhm and R. C. Davidson, "Theory of Free Electron Laser Instability in a Relativistic Annular Electron Beam", submitted for publication.
7. H. S. Uhm and R. C. Davidson, "Free Electron Laser Instability for a Relativistic Solid Electron Beam in a Helical Wiggler Field", submitted for publication.

DISTRIBUTION

	Copies
Naval Research Laboratory Attn: Dr. M. Lampe Washington, D. C. 20375	1
Office of Naval Research Attn: W. J. Condell (ONR-421), 800 N. Quincy St. Arlington, Virginia 22217	2
U. S. Army Ballistic Research Laboratory Aberdeen Proving Ground Attn: Dr. D. Eccleshall (DRDAR-BLB) Aberdeen, Maryland 21005	1
Air Force Weapons Laboratory Kirtland Air Force Base Attn: Maj. H. Dogliana Albuquerque, New Mexico 87117	1
Department of Energy Attn: Dr. T. Godlove (C-404) Washington, D. C. 20545	1
National Bureau of Standards Attn: Dr. J. M. Leiss Gaithersburg, Maryland 20760	1
Austin Research Associates, Inc. Attn: Dr. W. E. Drummond 1901 Rutland Drive Austin, Texas 78758	1
Ballistic Missile Defense Advanced Technology Center Attn: Dr. L. J. Harvard (BMDSATC-1) P. O. Box 1500 Huntsville, Alabama 35807	1
B. K. Dynamics, Inc. Attn: Dr. R. Linz 15825 Shady Grove Road Rockville, Maryland 20850	1

DISTRIBUTION (Cont.)

	Copies
The Charles Stark Draper Laboratory, Inc.	
Attn: Dr. E. Olsson	1
555 Technology Square	
Cambridge, Massachusetts 02139	
Director	
Defense Advance Research Projects Agency	
Attn: Dr. J. Mangano	1
1400 Wilson Boulevard	
Arlington, Virginia 22209	
IRT Corporation	
Attn: Mr. W. Selph	1
P. O. Box 81087	
San Diego, California 92138	
Los Alamos Scientific Laboratory	
Attn: Dr. G. Best	1
P. O. Box 1663	
Los Alamos, New Mexico 87545	
Mission Research Corporation	
Attn: Dr. C. Longmire	1
735 State Street	
Santa Barbara, California 93102	
Physical Dynamics, Inc.	
Attn: Dr. K. Breuckner	1
P. O. Box 977	
La Jolla, California 92037	
Sandia Laboratories	
Attn: Mail Services Section for:	1
Dr. R. B. Miller	
Albuquerque, New Mexico 87115	
Science Applications, Inc.	
Attn: Dr. M. P. Fricke	1
1200 Prospect Street	
La Jolla, California 92037	
Science Applications, Inc.	
Attn: Dr. R. Johnston	1
Dr. J. Siambis	1
2680 Hanover Street	
Palo Alto, California 94304	

DISTRIBUTION (Cont.)

	Copies
University of California Lawrence Livermore Laboratory	
Attn: Dr. R. J. Briggs	1
Dr. E. Lee	1
P. O. Box 808 Livermore, California 94550	
Defense Technical Information Center Cameron Station Alexandria, Virginia 22314	12
Naval Sea Systems Command Washington, D. C. 20362	
Attn: SEA-09G32	2
SEA-03B	1